DETERMINATION OF AERATION CAPACITY OF FERMENTERS WITH FAST RESPONDING OXYGEN PROBE AT HIGH RESPIRATION RATES

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The examination of oxygen probe response obtained at high values of respiration rate, when a linear oxygen concentration profile within the probe membrane at the beginning of aeration period I cannot be assumed, is presented. Method of calculation of aeration capacity from the response curve is given.

The method for determination of aeration capacity k_La and of respiration rate r in fermenters with an oxygen probe, proposed by Bandyopadhyay[1], is based on examination of the oxygen concentration changes observed after a sudden interruption (period II) and subsequent resumption (period I) of the gas supply to the fermenter (Fig. 1). The method cannot be used for determination of high values of k_La because the nonlinear deviation of oxygen probe reading at fast concentration changes of oxygen has been neglected. Heineken[2] derived a probe response for large values of aeration capacity assuming the batch of the fermenter free of oxygen at the beginning of the period I. Linek and coworkers[3] derived a normalised probe response for the case of sufficiently low values of respiration rate. On this assumption it was possible to use a linear concentration profile of oxygen within the probe membrane at the beginning of the period I.

In this communication the examination of the probe response obtained at high values of respiration rate is presented; it means the case is discussed where the linear concentration profile of oxygen within the membrane at the beginning of the period I cannot be assumed.

In our previous paper[3] the following relation for the oxygen concentration in a fermenter in the period II was derived:

$$c = c_2^+ - r/(k_La) - rt\,, \tag{1}$$

assuming gas and liquid phase in the fermenter perfectly mixed and $k_La = 0$ during the whole of the period II. We derived[3] the following relation for oxygen concentration profile in the probe membrane in the period II

$$c = \left(c_2^+ - \frac{r}{k_L a} - rt\right)\left(1 - \frac{x}{d}\right) + \frac{2r}{\pi k}\sum_{n=1}^{\infty}\frac{\sin(n\pi x/d)}{n^3}\left[1 - \exp(-n^2 kt)\right] \quad (2)$$

and the relation

$$\Gamma^r = 1 - \frac{r}{c - r/k_L a}\,t + \frac{r}{c_2^+ - r/k_L a}\frac{\pi^2}{6k} + \frac{2r}{c_2^+ - r/k_L a}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2 k}\exp(-n^2 kt)\,, \quad (3)$$

for the normalised probe reading Γ^r in this period. It follows from (3) that the value of respiration rate r equals to the slope of the chart of oxygen probe response, which is linear, if $kt \geqq 3{\cdot}7$. In this case the deviation of the slope from the value of r does not exceed 5 per cent. If the gas supply to the fermentor is interrupted for a time interval t_0 so short that $kt_0 < 3{\cdot}7$ then this simple method cannot be used. In Fig. 1 the oxygen probe response for $kt_0 \approx 3{\cdot}7$ is sketched. In this case the value of the respiration rate r can be calculated as the slope of the response curve at the end of the period II.

At the end of the period II the oxygen concentration profile in the probe membrane is given by (2) and (1) for $t = t_0$:

$$c = c_0\left(1 - \frac{x}{d}\right) + \frac{2r}{\pi k}\sum_{n=1}^{\infty}\frac{\sin(n\pi d/x)}{n^3}\left[1 - \exp(-n^2 kt_0)\right]. \quad (4)$$

An exact solution of the oxygen probe response in period I had to involve the oxygen profile (4) instead of the linear profile

$$c = c_0(1 - x/d)\,, \quad (5)$$

which was used in[3]. The analysis given there is correct as long as the sum of the infinite series in (4) may be neglected.

Using the following transformation

$$u = c - c_0\left(1 - \frac{x}{d}\right) - \frac{2r}{\pi k}\sum_{n=1}^{\infty}\frac{\sin(n\pi x/d)}{n^3}\left[1 - \exp(-n^2 kt_0)\right]\exp(-n^2 kt) \quad (6)$$

we convert the problem into solution of a diffusion equation $(\partial u/\partial t) = D(\partial^2 u/\partial x^2)$, with initial and boundary conditions: $u = 0$ for $x \leqq d$, $t = 0$; $u = 0$ for $x = d$, $t > 0$, $u = G(t) - c_0$ for $x = 0$, $t > 0$.

It was shown[3] that the oxygen concentration $G(t)$ in a fermenter as a function of time in the period I can be expressed

$$G(t) = \bar{c}^{+}[1 - \exp(-k_{L}at)] + c_0 , \tag{7}$$

where

$$\bar{c}^{+} = c_2^{+} - (r/k_{L}a) - c_0 . \tag{7a}$$

Further the development of the relation

$$M = \frac{k'D}{d}\left[c_0 - \frac{2r}{k}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}[1 - \exp(-n^2kt_0)]\exp\{n^2kt\} + \right.$$

$$\left. + \bar{c}^{+} - \frac{\bar{c}^{+}\,\pi B^{1/2}}{\sin(\pi B^{1/2})} - 2\bar{c}^{+}\sum_{n=1}^{\infty}(-1)^n\frac{\exp(-n^2kt)}{n^2/B - 1}\right] \tag{8}$$

for the probe response M, defined as

$$M = -k'D(\partial c/\partial x)_{x=d} \tag{9}$$

has been given. It follows from Eq. (8) that the value of oxygen concentration c_0^0, read by the oxygen probe at the moment of resumption of gas supply to the fermenter, is higher than the correct value c_0 at this moment. This deviation may be written as

$$e = c_0^0 - c_0 = \frac{2r}{k}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}[1 - \exp(-n^2kt_0)] . \tag{10}$$

The difference e is marked in Fig. 1.

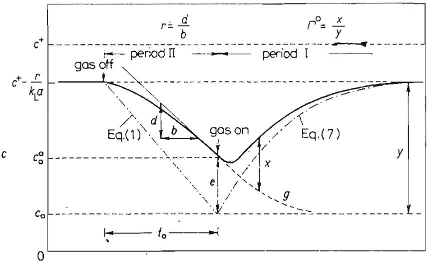

FIG. 1

Plot of Response Curves of the Probe Following Oxygen Concentration in a Fermenter after Sudden Stop of Gas Supply and of Agitator and after Subsequent Re-Opening of Gas Supply to the Fermenter

Let us define the following normalised response Γ^0:

$$\Gamma^0 = \frac{Md}{k'D\bar{c}^+} - \frac{c_0}{\bar{c}^+} + \frac{2r}{\bar{c}^+}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2k}\left[1 - \exp\left(-n^2kt_0\right)\right]\exp\left(n^2kt\right). \qquad (11)$$

Substituting (*11*) into (*8*) and comparing the result with the normalised response Γ derived in[3] for low values of respiration rate, it is obvious $\Gamma^0 = \Gamma$.

This finding allows us to say that the normalising method defined by (*11*) enables us to use the same relation for evaluation of aeration capacity k_La both for low and for high reaspiration rates. The values of Γ^0 can be calculated from the probe response recorded in the period I in the following way:

1. On using Eq. (*3*) determine the value of the respiration rate r from a pair of corresponding Γ^r and t taken from the record of the probe response in period II.

2. Calculate the value of z defined as the ratio of the probe reading situated in the fermenter liquid and in the gas phase leaving the fermenter in steady state:

$$z = (c_2^+ - r/k_La)/c_2^+ . \qquad (12)$$

3. Calculate the value of c_0 from Eq. (*1*):

$$c_0 = c_2^+z - rt_0 , \qquad (13)$$

where t_0 denotes the time interval of the interruption of gas supply to the fermenter.

4. Plot the curve g, given by the following equation, to the recorder chart (Fig. 1):

$$c = c_0 + \frac{2r}{k}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\left[1 - \exp\left(-n^2kt_0\right)\right]\exp\left(-n^2kt\right). \qquad (14)$$

5. One can calculate the values of Γ^0 as the ratio of the abscissae x and y read from the recorder chart (Fig. 1).

6. The aeration capacity k_La can be determined from a pair of Γ^0 and corresponding t values by the methods given in[3].

In order to illustrate the influence of the phenomena discussed let us present an example of enumeration of the value of e for operating conditions fairly current in fermenters. For the value of $k_La = 0{\cdot}3\ \mathrm{s}^{-1}$, $c_2^+ = 1{\cdot}3 \,.\, 10^{-4}\ \mathrm{kmol} \,.\, \mathrm{m}^{-3}$ and $z = 0{\cdot}8$ it follows from (*12*) $r = 7{\cdot}8 \,.\, 10^{-6}\ \mathrm{kmol\ m^{-3}\ s^{-1}}$. If we consider $c_0 = 0{\cdot}1c_2^+ = 1{\cdot}3 \,.\, 10^{-5}\ \mathrm{kmol\ m^{-3}}$ then t_0 can be calculated from (*13*): $t_0 = 11{\cdot}7\ \mathrm{s}$. As the constants k of currently used membranes are usually equal to about $0{\cdot}3\ \mathrm{s}^{-1}$ let us consider this value in our calculation. Putting these values into (*10*) we obtain for the difference $e = 4{\cdot}11 \,.\, 10^{-5}\ \mathrm{kmol\ m^{-3}}$. This value equals to 45 per cent of the full scale of oxygen concentration change y in fermentation broth.

Keeping in mind this high percentage the conclusion can be made that at high respiration rates the distortion of the response curves in period I is significant and the method developped for low respiration rates cannot be used. The method presented in this paper enables one to calculate the aeration capacity k_La of fermenters from response curves of oxygen probe obtained under high respiration rate conditions.

LIST OF SYMBOLS

a specific interfacial area
B $= K_La/k$
c oxygen concentration
c^+ equilibrium value of oxygen concentration
$\bar{c}^+$ concentration defined by *(7a)*
c_0 oxygen concentration in fermenter liquid at the end of period II
c_0^0 oxygen concentration reading of oxygen probe at the end of period II
d membrane thickness
D oxygen diffusion coefficient in membrane
e $= c_0^0 - c_0$
G function defined in the text
k membrane constant
k_L mass transfer coefficient
k' constant in *(9)*
M probe response defined by *(9)*
r respiration rate
t time
t_0 time interval of the interruption of gas supply to the fermenter
u transformation *(6)*
x space variable or abscissa defined in Fig. 1
y abscissa defined in Fig. 1
z quantity defined by *(12)*
Γ normalized response of oxygen probe in period I at low respiration rates
Γ^0 normalised response of oxygen probe in period I at high respiration rates
Γ^r normalised response of oxygen probe in period II

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2 output from the system

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